Visibility: How Applicable is the Century-Old Koschmieder Model?

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ABSTRACT

Koschmieder proposed that visibility is inversely proportional to the extinction coefficient of air, and this model has been widely adopted during the past century. Using radiative transfer theory, the authors present a general relationship for the law of contrast reduction and point out that the Koschmieder model is workable only to situations when a common-size object can be viewed tens of kilometers away. However, the Koschmieder model is not applicable for viewable distances of hundreds of meters when the angular dimension of an object is significantly greater than the eye resolution of the human being. The authors further separate the term ''visible'' into ''simple detection'' or ''detectability'' and ''clear identification'' or ''identifiability'' and point out that the Koschmieder model is applicable to identifiability, but not necessarily for detectability. In addition, the way of calculating contrast is revised to follow the concept of brightness constancy. The results of this effort advocate the measurement and distribution of detectability in harsh weather conditions, as such data offer more useful and important information for daily life.

1. Introduction

Our lives are strongly dependent on information, big or small, aged or new, because it forms the basis of sound decision-making. We use a suite of advanced technologies and instruments, from microscopic to macroscopic, to collect a broad range of information from human activities to the Earth system. However, our eye–brain system, the oldest ''imaging instrument,'' is still the most used and indispensable in our daily lives. With this system, events or objects are constantly observed and whether an object is visible or not impacts decision-making and management. Subsequently, the term ''visibility'' has been used to provide a quantitative representation of this information throughout the past decades.

However, there is no unified definition of visibility, although it is usually referred to as the distance of ''an object will be just *visible*'' (Duntley 1948b, p. 237; Malm et al. 1980; Middleton 1947), as adopted by the International Civil Aviation Authority (ICAA). On the other hand, the World Meteorological Organization (WMO) defines visibility as ''the length of path in the atmosphere required to reduce the luminosity of a collimated beam to 5% of its original value.'' Apparently, the former definition involves human perceptions, while the latter is simply a measure of atmospheric properties without human involvement. Consequently, the two visibility measurements may not provide consistent results.

During daylight hours (or the photopic regime), the visibility of an object to our eyes depends on a host of factors (Duntley 1948b). For instance, if an object is too small (i.e., smaller than the resolution of our eyes), then it is not perceivable. Similarly, if the object's color and brightness are similar to the background (e.g., creatures with camouflage capabilities), then it is also not (or hardly) visible. If the medium (air or water) is murky (e.g., dense fog or severe dust storms), then an object will also be imperceptibly visible even at close distances. With a focus on the quality of the medium and to have visual ranges observed at different times or at different locations comparable (Dabberdt and Eigsti 1981; Doyle and Dorling 2002; Ma et al. 2011; Majewski et al. 2015), visibility V_{cvt} (km) in the classical visibility theory

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(CVT) excludes situations of tiny targets and camouflages and, thus, takes the definition as that adopted by the ICAA (Duntley 1948a; Horvath 1981; Middleton 1947).

To explain the relationship between $V_{\rm cvt}$ and atmospheric properties, Koschmieder developed a simple theoretical model 90 yr ago [cited in Middleton (1947)], where $V_{\rm cvt}$ is inversely proportional to the extinction coefficient of the air e_a (km^{-1}); e_a is the sum of the absorption (a_a; km^{-1}) and scattering coefficients (s_a; km^{-1}) of the atmosphere (Middleton 1947). This relationship supports the visibility definition taken by the WMO, which is also the basis to use an instrument to measure visibility, although it actually measures, objectively, an air property (e_a) (Ahmed et al. 2014; Majewski et al. 2015; Marthinsen 2015). Subsequently, the visibility data provided by government agencies and weather forecasts are measured by visibility meters placed at various locations (e.g., airports, seaports, tourist sites, bridges, or highways).

Horvath (1971) reviewed this model 40 yr ago and concluded that the Koschmieder model "is only applicable under very limited conditions." Specifically, Horvath pointed out that the Koschmieder model requires the atmosphere to be illuminated homogeneously with a spatially constant atmospheric extinction coefficient and a black target viewed against the horizontal sky. Further, Horvath (1971) proposed a revised model and concluded as long as a reasonable black target and the averaged extinction coefficient are used, the Koschmieder model is accurate within ~10%.

Fundamentally, the word "visible" or "seen" is qualitative and thus inherently vague and subjective. There are at least two distinctive levels of seen in human visual decision processes (Zege et al. 1991): level 1 is the detection of an object, such as seeing a letter but being unable to distinguish it (i.e., is it a "C" or an "O"?). Level 2 is the identification of an object, such as distinguishing the letter "C" from letter "O." The level-1 seen focuses on the entire object and results in a "yes" or "no" binary answer, while the level-2 seen searches for much finer or detailed information. Unlike level 1, the answer for level 2 is not binary. Further, "identification" is a decision made after "detection" is achieved. For example, Fig. 1 shows the difference between detectable (large dark objects in a dust storm) and identifiable (windows of skyscrapers) objects under different atmospheric conditions.

Ideally we want to reach the level-2 seen or conclusion in visual ranging; however, in many cases, level-1 information may exist long before level 2 is reached. In other cases, level-1 information is also desired and an important source of knowledge (Duntley 1948b). For example, when sailing during fog, the information we may want to collect first is the location and distance of any boats (or obstacles) nearby rather than precise information such as the boat model. This is also true when we search for a known target in the field of view of our eyes, with the task of simply trying to locate its whereabouts, such as finding an airplane in the sky or observing a Secchi disk lowered into water. Further, for a highly scattering medium (e.g., fog, dust storms, turbid waters), an object quickly loses its sharpness owing to the effect of forward scattering (see Fig. 1 for an example). Thus, a primary task in such a situation is to achieve detection. Therefore, there are actually two distinct "visibilities" corresponding to the two levels of visual determination by our eye–brain system. To minimize the ambiguity associated with the term visibility, we here use "detectability" to represent the maximum distance of noticing an object (i.e., the distance before an object is no longer noticeable or detectable by our eye–brain system), while we use "identifiability" to represent the maximum distance of recognizing an object. It is thus necessary to assign a proper attribute—detectability or identifiability—to the visibility values predicted by the Koschmieder model.

In this article, after discussions regarding the theoretical derivations and assumptions associated with the CVT, we present a general relationship regarding contrast transmission based on radiative transfer and discuss the impact of the angular dimension of the observed target on contrast reduction. Subsequently, a general model of visibility consistent with the "brightness constancy" concept is developed. The results of this effort clarify and detail the concept of visibility and provide a unified model to interpret the visual ranging of both detection and identification.

2. Derivation and discussion of the Koschmieder visibility model

Let a target reside at a distance X (km) from an observer (Fig. 2). At a location x between the target and the eye, for a small distance dx (km), and under the assumption of no other light sources at this location, we have from radiative transfer theory (Chandrasekhar 1960; Duntley 1948a)

$$\frac{dL_T(x,\Theta)}{dx} = -e_a(x)L_T(x,\Theta) + \int_{4\pi} L_T(x,\Phi)\beta_a(x,\Phi)\,d\omega. \tag{1}$$

Here, $L_T(x, \Theta)$ is the radiance in the direction from the target to the eye, which contains both the information from the target and the radiance from ambient light; $L_T\,(x, \Phi)$ is the radiance within the 4π solid angle surrounding point x, with Φ measured relative to the direction of Θ; β_a is the volume-scattering function of the atmosphere (or the associated medium); and $d\omega$ is an infinitesimal solid angle (sr) at point x. Note that the

FIG. 1. Buildings in Beijing, China, observed from the same distance in (top) clear air and (bottom) a dust storm. When the air is clear, the windows of the buildings are visible, but they are not visible in a dust storm. Pictures courtesy of CNN Beijing Bureau.

equations and derivations will be the same if the light is measured in photometric units; therefore, for easier description and discussion, we adhere to radiometric quantities in this article.

For the radiance L_S from an adjacent location x' that is not part of the target but also propagates toward the eye, we have

$$\frac{dL_S(x',\Theta')}{dx} = -e_a(x')L_S(x',\Theta') + \int_{4\pi} L_S(x',\Phi)\beta_a(x',\Phi)\,d\omega. \quad (2)$$

Note that here $L_S(x', \Theta')$ is the radiance of the background (or reference) propagating toward the eye, with $L_S(x', \Phi)$ being the radiance in the 4π solid angle surrounding point x'.

Typically, the difference between $\beta_a(x)$ and $\beta_a(x')$ is negligible because of the closeness of the pair of points; subsequently, in CVT it is assumed that (Duntley 1948a, 1952; Preisendorfer 1986)

$$\int_{4\pi} L_T(x,\Phi)\beta_a(x,\Phi)\,d\omega = \int_{4\pi} L_S(x',\Phi)\beta_a(x',\Phi)\,d\omega. \quad (3)$$

Thus the following is derived from a simple subtraction between Eqs. (1) and (2) as the difference between $e_a(x)$ and $e_a(x')$ can also be considered negligible:

$$\frac{d[L_T(x) - L_S(x')]}{dx} = -e_a(x)[L_T(x) - L_S(x')]. \quad (4)$$

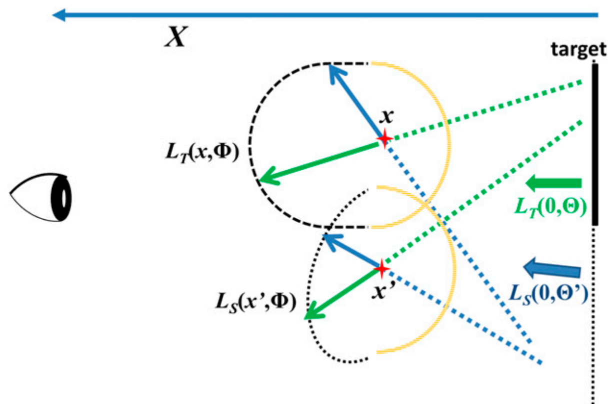

FIG. 2. An illustration comparing the light field between a point within the target x and a point outside of the target x'. Black dashed lines represent light in the hemisphere toward the eye, while yellow dashed lines represent light in the hemisphere toward the target; green arrows for radiance contain information from the target, while blue arrows are for radiance of the background. The arrow labeled X represents the distance between the target and the eye.

Note that the angular difference (Θ vs Θ') is omitted for brevity.

For simplicity, assuming a homogeneous atmosphere, integrating Eq. (4) from the target to the observer results in

$$L_T(X) - L_S(X) = [L_T(0) - L_S(0)]e^{-e_a X}, \tag{5}$$

where $L_T - L_S$ is the absolute contrast, or contrast in radiance units, between the target and the reference. For convenience of evaluation (Duntley 1948a; Horvath 1981; Middleton 1947), the contrast between the target and the background used in the CVT is defined as the relative difference between the light of the target and that of the background; that is,

$$C_a = \frac{L_T(X) - L_S(X)}{L_S(X)} \quad \text{and} \tag{6a}$$

$$C_i = \frac{L_T(0) - L_S(0)}{L_S(0)}, \tag{6b}$$

with C_i termed as the inherent contrast (i.e., no reduction by the medium) and C_a the apparent contrast perceived by the eye–brain system.

Substituting Eqs. (6a) and (6b) into Eq. (5) yields (Duntley 1948a; Middleton 1947)

$$C_a = \frac{L_S(0)}{L_S(X)} C_i e^{-e_a X}. \tag{7a}$$

For a distance X not too far apart (within tens of kilometers) between the target and the observer and assuming that the target is observed horizontally, $L_S(X)$ and $L_S(0)$ can be considered equal, and thus the above can be approximated as

$$C_a = C_i e^{-e_a X}. \tag{7b}$$

Equations (7a) and (7b) form the ''law of contrast reduction'' (Duntley 1948a, 1952; Preisendorfer 1986), which is the core component of the CVT that has been utilized in the past 90+yr for visibility studies (Aas et al. 2014; Duntley 1948a; Middleton 1947; Preisendorfer 1986; Zaneveld and Pegau 2003).

Visibility in the classical theory V_{cvt} is defined as the distance X, where C_a matches the threshold of the eye detection C_t; thus, V_{cvt} can be calculated as

$$V_{\mathrm{cvt}} = \frac{1}{e_a} \ln\left(\frac{C_i}{C_t}\right), \tag{8}$$

where C_t is generally considered as 0.02 (Middleton 1947), although a value of ~0.05 was later suggested (Dabberdt and Eigsti 1981; Gordon 1979). Note that C_t will become negative if the calculated C_i or C_a is negative (such as a black target in a bright sky) (Middleton 1947). For a perfect black target, $C_i = -1$, the above is reduced to

$$V_{\mathrm{cvt}} \approx \frac{3.9}{e_a}, \tag{9}$$

where 3.9 is $-\ln(0.02)$.

Equations (8) and (9) are the Koschmieder visibility model established 90 yr ago [cited in Middleton (1947) and Duntley (1948a)] and subsequently adopted for visibility research (Horvath 1981; Malm et al. 1980; Preisendorfer 1986). This model also serves as the theoretical base of the visibility meters manufactured and used today worldwide (Ahmed et al. 2014). The key assumption to reach Eqs. (4) and (5) is Eq. (3), but the validity of this assumption depends on the distance between x and x' used to evaluate the contrast, which is also critical for separating the level-1 or level-2 seen. As shown in Fig. 2, $L_T(x, \Phi)$ could include a large portion of light from the target; for $L_S(x', \Phi)$, however, the portion of light from the target could be much smaller. Only when the two points (x and x') are very close to each other will $L_T(x, \Phi)$ and $L_S(x', \Phi)$ nearly completely overlap, and then the assumption of Eq. (3) becomes valid. This is the case for identification, where the edge or outline of an object (e.g., the letters in the vision chart) is the target (Horvath 1981); thus, x and x' are side by side and therefore Eq. (3) is valid for such scenarios or objectives. This is also the case for a small target where the angular dimension of the target is similar to the angular resolution of our eyes (Middleton 1947).

For a large target with x at the center of the target (e.g., the large dark object in a heavy dust storm as shown in Fig. 1) and x' outside of the target, x and x' are far apart, and $L_T(x, \Phi)$ and $L_S(x', \Phi)$ will no longer be the same; thus, the assumption of Eq. (3) does not hold, and consequently Eqs. (4) and (5) and the law of contrast reduction cannot be derived. In essence, the Koschmieder model is applicable to small targets (edge or an object with small angular dimension) or identifiability, rather than for detectability of close-range or large targets.

This point can be further emphasized below. Mathematically, for Eq. (5) to be true for any X and e_a, we must have

$$L_T(X) = L_T(0)e^{-e_aX} + Y(X) \quad \text{and} \tag{10a}$$

$$L_S(X) = L_S(0)e^{-e_aX} + Y(X), \tag{10b}$$

with $Y(X)$ as the background. For Eqs. (10a) and (10b) to be true, the target must be a point source for our eye–brain system (i.e., an angular dimension comparable to the angular resolution of our eyes).

For visual ranging in a scattering medium, such as sailing in fog or observing a Secchi disk lowered into water, the edge of an object, which is an abstract concept, is lost quickly (see Fig. 1 for an example) and identifiability is a secondary objective compared to detectability (i.e., the level-1 seen). For such level-1 visual ranging, as it is a yes or no binary answer, the judgment is no longer determined by focusing on the edge of the object but rather by spotting any portion of the target (see Fig. 1 for buildings in a dust-storm day). The distance between x and x' for such simple detections is then no longer fixed; it rather depends on the relationship between the angular dimension of the target ψ_T and the angular resolution of our eyes ψ_e. Note that, in addition to the superspectral resolution and superdynamic range, our eyes also have an extraordinary angular resolution [~0.0003 rad (Clark 1990; Curcio et al. 1990)]. This resolution transfers to a spatial resolution of ~0.3 mm at a distance of 1 m (Curcio et al. 1990) (https://en.wikipedia.org/wiki/Naked_eye). Therefore, for an object (say, a boat) 6 m in size in a fog and assuming a maximum range of sighting this target of just 100 m, its ψ_T is ~0.06 rad, which is 200 times greater than ψ_e. For such cases, this eye imager no longer measures light from a point source for identification, but collects light from a broad source (the boat); consequently, the reduction of L_T with the increase of distance no longer follows the beam attenuation coefficient as described by Eq. (10a) (also see section 3). It is therefore clear that the Koschmieder model [and the revised formula by Horvath (1971)] is generally appropriate for identifiability (i.e., a clear recognition of small objects). These features indicate that the visibility data measured based on Eq. (8) in a fog or dust storm are not truly the maximum distance of detecting an object with common sizes (such as a building in dust storm; see Fig. 1), although they provide a measure for potentially discerning fine details (such as the windows in dust storm; see Fig. 1) or the so-called meteorological visibility.

3. Law of contrast reduction of any size targets

For both $L_T(x, \Phi)$ and $L_S(x', \Phi)$ in Eqs. (1) and (2), we may divide them into two portions, respectively, with one portion propagating toward the observer side [$L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$; the black dash lines in Fig. 2], the other portion propagating toward the target side [$L_{Tt}(x, \Phi)$ and $L_{St}(x', \Phi)$; the yellow dash lines in Fig. 2], then Eqs. (1) and (2) can be written as

$$\frac{dL_T(x)}{dx} = -e_aL_T(x) + \int_{2\pi_o} L_{To}(x,\Phi)\beta_a(\Phi)\,d\omega + \int_{2\pi_t} L_{Tt}(x,\Phi)\beta_a(\Phi)\,d\omega \quad \text{and} \tag{11a}$$

$$\frac{dL_S(x')}{dx} = -e_aL_S(x') + \int_{2\pi_o} L_{So}(x',\Phi)\beta_a(\Phi)\,d\omega + \int_{2\pi_t} L_{St}(x',\Phi)\beta_a(\Phi)\,d\omega. \tag{11b}$$

Here, $2\pi_o$ and $2\pi_t$ are the solid angles of the hemisphere of the observer and target sides, respectively, and Θ and Θ' are omitted for brevity. Further, for brevity and without losing generality, it is assumed that the atmosphere is homogeneous (i.e., e_a and β_a are independent of location). Because $L_{Tt}(x, \Phi)$ and $L_{St}(x', \Phi)$ are a pair of ambient light propagating toward the target side, the difference between the two can be considered negligible (although may not be exactly so for positions close to the target and under dust storms or foggy conditions owing to strong multiple scattering); hence, we have

$$\frac{d[L_T(x) - L_S(x')]}{dx} = -e_a[L_T(x) - L_S(x')] + \int_{2\pi_o} [L_{To}(x,\Phi) - L_{So}(x',\Phi)]\beta_a(\Phi)\,d\omega. \tag{12}$$

The difference between $L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$ depends on the distance between x and x'. As discussed earlier, for a target with a small ψ_T or adjacent pairs at the target edge, $L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$ nearly overlap each other and the difference approaches 0. For a broad target ($\psi_T \gg \psi_e$) with location x pointing to the center of the target, while x' is outside of the target, then the difference between $L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$ could be large, especially if the target is much brighter than the background and location x is near the target. Following these general considerations and for easy illustration, we may rewrite Eq. (12) as

$$\frac{d[L_T(x) - L_S(x')]}{dx} = -e_a[L_T(x) - L_S(x')] + \varepsilon [L_T(x) - L_S(x')] \int_{2\pi_o} \beta_a(\Phi)\, d\omega, \quad (13)$$

with ε defined as

$$\varepsilon = \frac{\int_{2\pi_o} [L_{To}(x,\Phi) - L_{So}(x',\Phi)]\beta_a(\Phi)\, d\omega}{[L_T(x) - L_S(x')]\int_{2\pi_o} \beta_a(\Phi)\, d\omega}. \quad (14)$$

Parameter ε basically represents the relative relationship between $L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$, which will be a function of ψ_T and the ambient light. The detailed dependence of ε on both ψ_T and ambient light is beyond this effort. However, intuitively and following Wells (1973), for a point source (small ψ_T), $L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$ overlap causing ε to approach 0; while for a broad target ($\psi_T \gg \psi_e$) with an isotropic light source, $L_{To}(x, \Phi)$ and $L_{So}(x', \Phi)$ are independent of angle, and ε will approach 1. The integration of β_a in $2\pi_o$ is the forward-scattering coefficient of the air s_{af} (km^{-1}), thus, Eq. (13) can be written as

$$\frac{d[L_T(x) - L_S(x')]}{dx} = -\kappa_a(x)[L_T(x) - L_S(x')], \quad (15)$$

with κ_a (km^{-1}) the effective attenuation coefficient of the medium, which is

$$\kappa_a = e_a - \varepsilon s_{af}. \quad (16)$$

Therefore, as shown in Lee et al. (2015) for underwater visibility, a general relationship for contrast transmission through a homogeneous medium is

$$L_T(X) - L_S(X) = [L_T(0) - L_S(0)]e^{-\kappa_a X}. \quad (17)$$

For nonhomogeneous air properties, κ_a will simply be a spatially averaged value (Horvath 1971). Thus Eq. (17) expands upon Eq. (5) in describing the reduction of contrast through a medium. For targets with a small ψ_T (identifiability), ε approaches 0 and κ_a approaches e_a, then Eq. (17) reduces to the traditional Eq. (5). For observing a broad target (detectability), ε approaches 1 and κ_a is reduced to

$$\kappa_a \approx a_a + s_{ab}, \quad (18)$$

with s_{ab} (km^{-1}) the backscattering coefficient of the air. This transition of the effective attenuation coefficient between small and large targets is consistent with the transmission of image quality where a high-frequency image (small target) attenuates following the beam attenuation coefficient (Hou et al. 2007; Zege et al. 1991), while a low-frequency image (broad target) attenuates following the sum of the absorption and backscattering coefficients (Wells 1973). Note that s_{af} can be significantly larger than s_{ab} for aerosols or fog (Xu et al. 2015); thus, e_a can be much greater than $a_a + s_{ab}$, and the detectability is significantly longer than the identifiability. This is exactly what we experienced in visual ranging, where we have to be much closer to the object to observe fine details (the level-2 seen) although we can notice this object quite farther away (the level-1 seen). This is further evidenced with the various targets in Fig. 1, where the larger targets (e.g., the large buildings) are visible in the dust storm, while the smaller targets (e.g., windows in the buildings) are not visible. However, present visibility meters used worldwide measures value of e_a; thus, the converted visibility from this e_a data does not necessarily represent the detectability in harsh weather conditions (e.g., fog and dust storms) where the viewable distance is in the hundreds of meters or less. There is an urgent need to manufacture instruments to provide true measurements of visibility for such conditions in order to improve decisions and managements related to our daily lives.

In addition, the change of the effective attenuation coefficient between small and large targets explains the decrease of the detection threshold for larger targets shown in Blackwell (1946). It is interesting that Duntley (1948a) intuitively obtained a similar expression as Eq. (18), but mysteriously a model like this was largely ignored by the community. This may be partially due to the ambiguity of the definition of seen in studies of visibility.

4. Description of "contrast"

The contrast evaluated by Eq. (5) or (17) is an absolute contrast with radiometric or photometric units. Conventionally, it is the relative contrast [or the commonly termed contrast; Eq. (6)] used in the CVT. This relative contrast defined by Eq. (6) is a useful quantity to describe the sharpness of an image. However, this

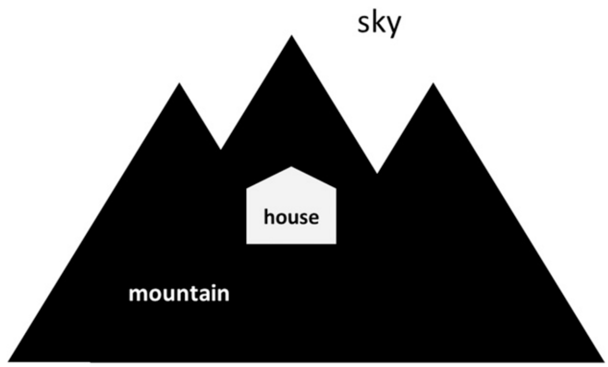

FIG. 3. A hypothetical case where a gray house is superimposed on a black mountain under an overcast sky. Following the classical visibility theory, we may not be able to detect the house using the sky as the background (reference), but we should be able to notice it using the mountain as the background even under dense fog and hundreds of kilometers away as long as the house is large enough in size.

definition of contrast is for convenience (Horvath and Noll 1969; Middleton 1947), rather based on principles of visual decision or judgment. Our eye–brain system utilizes differences in brightness or radiance, rather than the relative difference in brightness, to make judgment decisions (Blackwell 1946). In addition, this calculation of relative difference is subjective, as there is no reason for using L_S, rather than L_T, as the denominator for the calculation of this relative contrast. Indeed, some researchers (Hou et al. 2007; Zege et al. 1991) used the average of L_T and L_S as the denominator to avoid a subjective selection for this calculation.

Further, using Eq. (6) to calculate contrast could result in contradictions in theoretically interpreting visual ranging. Hypothetically, let us consider that there is a gray house on a black mountain under an overcast sky (Allard and Tombach 1981) (Fig. 3); the background (or reference) for this target (house) could then be either this black mountain or the gray sky. If using the sky as the background, the relative contrast between the house and the sky may not be strong enough for us to detect the existence of this house (the level-1 seen) based on Eq. (6). However, if the black mountain is used as the background, the relative difference by Eq. (6) will be infinite (Duntley 1948b); thus, we are able to detect this house, and this visual perception is consistent with our experiences (target with a shadow has greater visibility, for instance). On the other hand, based on Eq. (8), because C_i is infinite with the black mountain as the background, this house should then be detectable even if it is thousands of kilometers away and under a dense fog, as long as its ψ_T is greater than ψ_e. This is not consistent with our experiences. We thus see ambiguous results from Eq. (8) for such a scenario. There should be just one answer on sighting the house and the target could not be detected hundreds of kilometers away in a heavy fog.

As highlighted in Blackwell's experiment (Blackwell 1946), the detection of a target by our eye–brain system depends on the maximum detectable difference in light. For the case of the house–black mountain–cloudy sky, our eye–brain system will use the maximum difference in brightness (between the house and the black mountain in this case) for this detection. On the other hand, the detection threshold in light intensity of our eye–brain system is not fixed, but changes with the ambient light that our eyes are exposed to (Blackwell 1946). Taking into account the adjustment of our eye–brain system, a more appropriate contrast for target detection can be defined as

$$\chi = \frac{L_T(X) - L_S(X)}{E(X)}, \tag{19}$$

with E representing the irradiance (or illuminance in photometric domain) that our eyes are exposed to in the photopic regime. In such a definition for the relative contrast, E will not be dependent on the selection of the background (or reference) and this definition is consistent with the brightness constancy concept in visual perception (Freeman 1967).

Applying Eq. (17), we get

$$\chi = \frac{L_T(0) - L_S(0)}{E(X)} e^{-\kappa_a X}. \tag{20}$$

At the time the target disappears from sight, $E(X)$ basically represents the ambient light from both the sun and sky (assuming no other light sources in the field of view of the eyes), and $E(X)$ can be estimated from $E(0)$ (the total irradiance at the target location) given their locations and atmospheric properties. Note that the ratio $L_T(0)/E(0)$ (represented as R_T, in the following) and the ratio of $L_S(0)/E(0)$ (represented as R_S in the following) represent the reflectance of the target and the background, respectively; thus, Eq. (20) can be written as

$$\chi = \frac{E(0)}{E(X)} (R_T - R_S) e^{-\kappa_a X}. \tag{21}$$

Equation (21) then serves as a general relationship of the contrast reduction of any size targets adjusted to the sensitivity of our eyes. Since visibility is the distance where the apparent contrast χ approaches the detection threshold of our eyes τ, there is

$$V_{\mathrm{nm}} = \frac{\Gamma}{\kappa_a}, \qquad (22)$$

with Γ as

$$\Gamma = \ln\left[\frac{E(0)}{E(V_{\mathrm{nm}})}\frac{R_T - R_S}{\tau}\right]. \qquad (23)$$

Here, V_{nm} (km) is the visibility based on the new model for contrast reduction and Γ is a parameter associated with both the inherent contrast of the target and the eye threshold τ measured in the reflectance domain. A value of ~0.013 sr^{-1} for τ is found reasonable for a circular white disk (Lee et al. 2015), but more studies, especially for harsh weather conditions, are necessary to determine the values of τ. Note that for a target viewed horizontally, the difference between $E(0)$ and $E(V_{\mathrm{nm}})$ should be negligible for homogeneous medium and for V_{nm} within tens of kilometers; thus, the ratio of $E(0)/E(V_{\mathrm{nm}})$ becomes 1.

Equations (22) and (23) serve as a general model for visual ranging, either for detectability or identifiability by our eye–brain system, in any medium. The rules can be summarized as follows:

1) Both detectability and identifiability depend on the difference of reflectance between the target and the background, which is consistent with the brightness constancy concept in visual perception; this is also the principle of camouflage.
2) If there are multiple references in the scene, V_{nm} is determined by the maximum difference in reflectance, and this is why a target with a shadow has greater visibility than a target without a shadow (Allard and Tombach 1981; Gordon 1979).
3) Because R_S is azimuth dependent, V_{nm} is also azimuth dependent (Middleton 1947).
4) For identifiability where ψ_T is just slightly greater than ψ_e, V_{nm} is inversely proportional to e_a ($=a_a + s_a$) (i.e., the Koschmieder model established 90 yr ago).
5) For detectability, because κ_a depends on the angular dimension of an object, the value of κ_a is not a constant even for homogeneous atmospheric properties. This is supported by Fig. 1 where some targets or objects (e.g., the large buildings) can be detected in the heavy dust storm, but some (e.g., the windows of those buildings) are not. Further, for common objects such as an airplane or a boat, usually sized at meters or tens of meters, the maximum distance to detect such objects is tens of kilometers when observed in clear sky days (Horvath and Noll 1969), thus $\psi_T \sim \psi_e$, and the detectability in such situations is inversely proportional to e_a (i.e., the Koschmieder model). However, when such objects are observed in a fog or dust storm, $\psi_T \gg \psi_e$, the viewable range is in the tens of meters, and the detectability is inversely proportional to $e_a - \varepsilon s_{af}$, instead of e_a. Therefore, the visual ranging estimated with the Koschmieder model will thus be significantly shorter than the actual detectable distance for such situations. On the other hand, because of the advancement in electro-optical systems, visual ranging of targets with a distance of tens of kilometers by a human eye is no longer necessary or important. It is the detectability of close-range targets, such as cars or ships in foggy or dust-storm days, that is desired and useful for our daily lives.

Note that in the above the halo effect of an object (Aas et al. 2014; Preisendorfer 1986), a result of multiple scattering between the photons from the target and photons of the background is omitted. Because of this extra contrast information between the target and the background, the actual detectability by our eye–brain system will be slightly longer than that predicted by Eq. (22). On the other hand, because such effects are usually associated with strong scattering medium, this effect further suppress our ability to clearly identify an object (Hou et al. 2007; Zege et al. 1991), as we usually experience in foggy days.

5. Summary

Depending on the objective of visual ranging, there are at least two levels of ‘‘seen’’ for an object: one is ‘‘simple detection’’; the other is ‘‘clear identification.’’ Because of the different attenuation coefficients of the contrast transmission associated with the two visual perceptions, which are the result of the different relationships between ψ_T and ψ_e, there are also two different ‘‘visibilities’’: ‘‘detectability’’ or ‘‘identifiability.’’ Radiative transfer modeling suggests that the Koschmieder model is workable to identifiability or for identification of fine details (i.e., $\psi_T \sim \psi_e$). However, for simply detecting a large object (i.e., $\psi_T \gg \psi_e$), the detectability in air is proportional to the inverse of the backscattering coefficient (when the absorption coefficient is very small and ignorable). Consequently, the ‘‘visibility’’ value predicted with the Koschmieder model will not match the perception of our eyes, and the actual range of detection will be significantly longer than the visibility estimated from the Koschmieder model.

In addition, the visibility meters used globally are manufactured based on Eq. (8), where the actual measured property is the extinction coefficient e_a. Thus, for detectability, a new system is required to provide visual ranging for commonly encountered objects (e.g., cars, ships), especially under harsh weather conditions such as fog or dust storms.

Historically, contrast has been evaluated as the relative difference between the light associated with the target and the light from the background, which does not coincide with the visual perception process of our eye–brain system. Based on the ''brightness constancy'' concept, contrast is better evaluated as the difference in reflectance between the target and the background (or reference), and if this difference is great enough, the target will be detectable in the photopic regime regardless of the intensity of the incident light.

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